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There has been recent interest in using short-pulse radar to detect targets in lossy clutter. The analysis presented here shows that the energy and peak-power densities of pulses decay exponentially with depth in homogeneous, lossy, dispersive materials, provided the frequency bands of the pulses are separated from DC. Many numerical examples verify the analytical results.

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Radiated pulses decay exponentially in materials in the far fields of antennas

T.M. Roberts

There has been recent interest in using short-pulse radar to detect targets in lossy clutter. The analysis presented here shows that the energy and peak-power densities of pulses decay exponentially with depth in homogeneous, lossy, dispersive materials, provided the frequency bands of the pulses are separated from DC. Many numerical examples verify the analytical results.

Introduction: Pulse decay in Lorentz and Debye dispersion models has been studied [1–4] since 1914. Such pulses are said to decay algebraically with depth, typically as $x^{-1/3}$, $x^{-1/2}$, or $x^{-2/3}$ as $x \to \infty$ in these lossy models. This is slower than the exponential decay of single-frequency signals. Several groups recognised this and recently began investigating whether algebraically decaying pulses could usefully penetrate trees or other far-field, lossy clutter in front of targets. As explained in this Letter, it is concluded that the answer is, unfortunately, no.

Algebraic decay is often claimed for pulses with DC or near-DC content. Endnote 30 of [4], e.g. states that a pulse $f(t)$ decays as $x^{-1/3}$ in all undamped Lorentz models if f has DC content; but that the decay is $x^{-2/3}$ if the Fourier transform satisfies $\tilde{f} = 0 \neq d\tilde{f}/d\omega$ at $\omega = 0$, signifying near-DC content. Gauss-modulated cosines ($x^{-1/3}$) and sines ($x^{-2/3}$) are examples. Such pulses propagate well in co-axial cables; however, it is widely known that highly conducting ($0 < \omega\epsilon_0 \ll \sigma < \infty$) dipoles and loops have free-space radiation efficiencies that vanish as $\omega^{3/2}$ and $\omega^{7/2}$, respectively, as $\omega \to 0$. To model radar penetration of far-field clutter, it is therefore assumed here that the pulse spectrum is separated from DC.

Analysis: Let f be any real-valued, band-limited, incident electric-field pulse. Then f propagates in any 1D dispersive half-space $x \geq 0$ as

$$E(x, t) = \int_{-\omega_{max}}^{\omega_{max}} e^{i[k(\omega)x+\omega t]}\tilde{f}(\omega)d\omega \qquad (1)$$

with $k = k_r + ik_i = \omega[\epsilon(\omega)]^{1/2}/c$. Here $\tilde{f}(\omega) = 0$ except where ω satisfies $0 < \omega_{min} \leq |\omega| \leq \omega_{max} \leq \infty$. (A relevant fine point is considered below.) The material is lossy: $k_i > 0$ in $[\omega_{min}, \omega_{max}]$, with extreme values k_i^{min} and k_i^{max}. Standard analysis and (1) yield

$$|E(x, t)| \leq \exp(-k_i^{min}x)\int_{-\infty}^{\infty} |\tilde{f}|d\omega$$

i.e. $|E|$ decays at least as fast as $\exp(-k_i^{min}x)$. The Parseval equation similarly yields

$$\int_{-\infty}^{\infty} |E|^2 dt = \int_{-\infty}^{\infty} |\exp(ikx)\tilde{f}|^2 d\omega \leq \exp(-2k_i^{min}x)\int_{-\infty}^{\infty} |f|^2 dt$$

Thus, energy densities and peak $|E|$ values decay exponentially in all lossy, dispersive materials.

For example, every finite-band pulse separated from DC will decay exponentially in every Debye and damped-Lorentz model. Similar behaviour occurs in the loss bands $b^2 < \omega^2 < a^2 + b^2$ of undamped-Lorentz models $\epsilon = 1 + a^2/(b^2 - \omega^2)$. These examples are unlike the algebraic decay predicted [1–4] for near-DC-content pulses in the same Debye and Lorentz models.

Regarding fine points, the mathematics of entire functions shows that no finite-bandwidth pulse is precisely 0 over any time interval. Yet the results above do hold if $\omega_{max} < \infty$. Finite bands are also practical, and their never-precisely-0 consequences suggest ordinary noise. Indeed, the numerics here will omit features 55 dB below peak power. 'Power' and 'energy' hereafter refer implicitly to densities.

Numerics: The example incident pulses have $\tilde{f}(\omega) = 0$, except for $\omega_{min} \leq |\omega| \leq \omega_{max}$ where $\tilde{f} = \exp\{1 + \omega_b^2/[(|\omega| - \omega_c)^2 - \omega_b^2]\}$ with $\omega_{min} = \omega_c - \omega_b$ and $\omega_{max} = \omega_c + \omega_b$. The computed inverse transform of $\tilde{f}$ is truncated below −55 dB. Conveniently for numerics, this f is briefer than many other pulses with similar spectra. Fig. 1 shows the f used in Fig. 2 for a Debye model. Every f used here for a Lorentz model is in the spectrum of that model's so-called Brillouin precursors, which are often said to decay algebraically.

The Debye model $\epsilon_D = 1 + 58/(1 - i\omega \times 9.4\,\text{ps})$ approximates water. Let $u = 1$ rad/s. The damped-Lorentz model $\epsilon_L = 1 + 39 \times 10^{22}u^2/(16 \times 10^{22}u^2 - \omega^2 - 5i\omega \times 10^{10}u)$ is used so ϵ_D and ϵ_L have comparable $k_i(\omega)$ curves in Fig. 3. Indeed, Fig. 3 shows that the bands 13–18 GHz in ϵ_D and 40–50 GHz in ϵ_L have the same k_i^{min} and k_i^{max}. Propagating the corresponding pulses in ϵ_D and ϵ_L yields normalised energies $\mathcal{E}(x) = [\int |E(x, t)|^2 dt]/[\int |f|^2 dt]$ in ϵ_D and ϵ_L that overlap within a line width in the centre of Fig. 2. Normalised peak powers $\mathcal{P}^2(x) = \max_t |E(x, t)|^2/\max_t |f|^2$ in ϵ_D and ϵ_L also overlap $\mathcal{E}$ below $\exp(-2k_i^{min}x)$ in Fig. 2, verifying the analytical results.

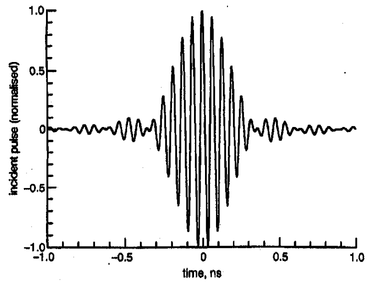

Fig. 1 *Near-peak values of an incident f(t) with parameters $\omega_{min}/2\pi = 13.48$ GHz and $\omega_{max}/2\pi = 18.01$ GHz*

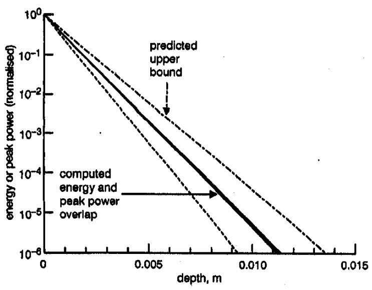

Fig. 2 *Normalised energies and peak powers of a pulse in ϵ_D and another in ϵ_L overlap (centre), are $\leq \exp(-2k_i^{min}x)$ (top), and are $\geq \exp(-2k_i^{max}x)$ (bottom)*

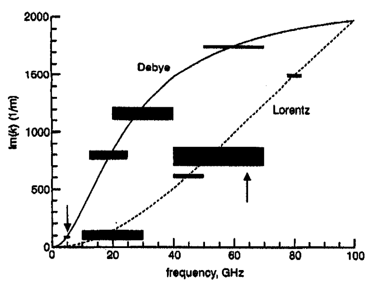

Fig. 3 *Curves $k_i(\omega)$ for models ϵ_D and ϵ_L, with exponential decay of peaks verified by eight examples described in text regarding rectangles*

Going beyond the analysis, a more sensitive measure of exponential decay is $-\mathcal{E}'/\mathcal{E} = -d[\ln \mathcal{E}(x)]/dx$. This is -1 times the x-dependent slope (in dB/cm) of $\mathcal{E}(x)$ when graphed in dB. Thus, $-\mathcal{E}'/\mathcal{E}$ is the local exponential decay rate of $\mathcal{E}$. For the pulses of Figs. 1 and 2, the computed $-\mathcal{E}'/\mathcal{E}$ varies from 1185–1257/m (51–55 dB/cm) for the first 60 dB of energy attenuation. This is in the range $[2k_i^{min}, 2k_i^{max}]$ of 1016–1485/m (44–64 dB/cm) suggested but not yet predicted by theory.

Fig. 3 shows ranges of local exponential decay rates $-\mathcal{P}'/\mathcal{P}$ of peaks $\mathcal{P}$ for the first 60 dB of peak-power attenuation. The large, marked rectangle, e.g. signifies that the incident f with spectrum 40–70 GHz has a peak that decays in the Lorentz model ϵ_L at an exponential rate that varies from 709–868/m. The dashed curve shows this is within the range $[k_i^{min}, k_i^{max}]$ of 508–1247/m for ϵ_L. This example and the seven others in Fig. 3 verify the $\leq \exp(-k_i^{min}x)$ prediction for peaks. The spectrum for the small, marked rectangle is 4–6 GHz. Linearity and Fig. 3 thereby yield two four-parameter families of examples of peaks that decay faster than $\exp(-k_i^{min}x)$, with bandwidths over three and four octaves.

Conclusions: In this Letter a practical model of pulses in far-field, lossy materials is used to show that exponential decay is typical. This is verified by many numerical examples, with bandwidths up to four octaves. Two other numerical observations have not yet been explained by analysis. First, analysis has not explained why $\exp(-2k_i^{max}x)$ is a bound in Fig. 2. Secondly, the local exponential decay rates of energies and peaks are within bounds suggested, but not yet predicted, by analysis. The analytical results in this Letter, however, are all numerically verified.

A consequence of recent, practical interest is that algebraic decay no longer seems to be a useful design principle for radar penetration of far-field, lossy clutter. Although algebraic decay might be recovered from exponential decay in a mathematical limit $\omega_{min} \to 0$, this would not escape the real difficulties of low radiation efficiency and low resolution posed by near-DC signals.

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References

1 BRILLOUIN, L.: 'Wave propagation and group velocity' (Academic Press, 1960)
2 OUGHSTUN, K.E., and SHERMAN, G.C.: 'Electromagnetic pulse propagation in causal dielectrics' (Springer-Verlag, 1994)
3 KELBERT, M., and SAZONOV, I.: 'Pulses and other wave processes in fluids' (Kluwer Academic, 1996)
4 ROBERTS, T.M., and PETROPOULOS, P.G.: 'Asymptotics and energy estimates for electromagnetic pulses in dispersive media', *J. Opt. Soc. Am. A*, 1996, 13, (6), pp. 1204–1217